START Selected Topics in Assurance Related Technologies

Volume 7, Number 4

Analysis of “One-Shot” Devices

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Introduction

A “one-shot” device is defined as a product, system, weapon, or equipment that can be used only once. After use, the device is destroyed or must undergo extensive rebuild. “One-shot” devices typically spend their life in dormant storage or stand-by readiness. The device may end its useful life without ever being called upon to provide the function for which it was designed, limiting the availability of failure data during its life cycle.

Determining the reliability of a “one-shot” device poses a unique challenge to the manufacturers and users of these devices. Due to the destructive nature and costs of the testing, the current trend is to minimize testing. But the expectations are to have a high level of system reliability. Therefore, the test planner must have the knowledge necessary to determine the minimum sample size that must be tested to demonstrate a desired reliability of the population at some acceptable level of confidence.

This START sheet addresses the steps necessary to statistically establish the reliability, or probability of success, of “one-shot” devices.

Background and Concepts

Statistical tools are designed to analyze the distribution characteristics of some population based on a sample drawn from the population. For “one-shot” devices, acceptance sampling is a statistical method used to predict the probability of success, or reliability, by estimating an attribute of the population through a sample. An attribute is an inherent characteristic that is evaluated in terms of whether or not the product performs as designed. Test results are measured by determining if the product was good or bad, passed or failed, etc. Non-conformance of the product characteristic is generally expressed as a proportion defective. Proportion defective is the number of failures that occurred in a sample size divided by the sample size.

Attribute sampling uses the binomial equation to test a hypothesis that a product has an acceptable defective rate at some acceptable level of risk. For “one-shot” devices, the object is to verify that the probability of success, when the device is called upon to function, is satisfactory at some desired level of confidence.

Binomial Distribution

The binomial distribution is based on the work of Jacob Bernoulli (1654 – 1705). The distribution is based on “Bernoulli trials”, where each trial will result in only of two possible outcomes, i.e., passed or failed. To use the binomial distribution to predict the probability of success for “one-shot” devices, the trials in the sample must meet the following conditions:

- Each trial must be independent. The outcome of one trial cannot influence an outcome of another trial.
- For each trial, there is only one of two possible outcomes.
- The number of trials in a sample must be fixed in advance and be a positive integer number.
- The probability of success must be the same for all trials.

The binomial equation to predict the probability of a specific number of r defects or failures in n samples is:

$$P(r) = \frac{n!}{r!(n-r)!} p^{r} (1-p)^{(n-r)} \qquad (1)$$

where:

p = proportion defective
n = sample size
r = number defective
P(r) = probability of getting exactly r defective or failed units in a sample size of n units

The desired proportion defective is the Lot Tolerance Percent Defective (LTPD), which is the poorest quality in an individual lot that one is willing to accept.

To calculate the probability of k or fewer failures occurring in a test of *n* units, the probability of each failure occurring must be summed, as shown in Equation 2.

$$P(r \leq k) = \sum_{r=0}^{k} P(r) \tag{2}$$

The Confidence Level that the population is only p defective based on r ≤ k defects from a sample of n is:

$$\text{Confidence Level} = CL = 1 - P\,(r \leq k) \tag{3}$$

For example, assume that the population of a part can be no more than 10% defective ($p = 0.1$). The plan is to test twenty parts and allow only one failure (pass-fail criterion). Using Equation 1, the probabilities of exactly one and exactly zero failures occurring are:

$$P(r = 0) = 0.122$$
$$P(r = 1) = 0.270$$

Using Equation 2, the probability of one failure or less is the sum of these probabilities; i.e.,

$$P(r \leq 1) = 0.122 + 0.270 = 0.392$$

Using Equation 3, if the sample passes the test, one would only be 60.8% confident ($CL = 1 - 0.392 = 0.608$, or 60.8%) that the proportion defective in the population is 10% or less.

The sampling plan is inadequate for us to be 90% confident that the population is no more than 10% defective. The sample size must be increased, the number of allowable failures decreased, or both. Using Table 1, when one failure is allowed, the sample size must be at least 38 for us to be 90% confident that the population is no more than 10% defective.

If no failures are allowed in the 20 tests, $P(r = 0) = 0.122$, the confidence level increases to 87.8% that the proportion defective in the population is 10% or less. To reach a 90% CL, the sample size would have to be 22 with no failures allowed. Table 1 shows the relationship between the number of failures and sample size for tests to demonstrate that the proportion defective in the population is 10% or less at Confidence Levels of 90% and 95%.

Estimating p

Assume a sample of 38 units was tested and two failures occurred. The proportion defective of the population can be estimated by calculating the upper and lower confidence limits of the true p for the population from which the sample was drawn. To do this, we use the F distribution.

Table 1. Failures Allowed vs. Sample Size vs. Confidence Level (CL) for 90% Reliability (10% defective rate)

No. of Failures	Sample Size	
	90% Confidence	95% Confidence
0	22	29
1	38	47
2	52	63
3	65	77
4	78	92
5	91	104
6	104	116
7	116	129
8	128	143
9	140	156
10	152	168

Equations 4 and 5 show how the lower (p_L) and upper (p_U) limits on p are calculated.

$$p_L = \frac{1}{1 + \left[(n - r + 1)/\,r\right] F_L} \tag{4}$$

where:

r is the number of failures observed
n is the sample size

F_L corresponds to the F distribution for the following degrees of freedom and associated required CL

$$\nu_1 = 2\,(n - r + 1)$$
$$\nu_2 = 2\,r$$

$$p_U = \frac{1}{1 + \dfrac{n - r}{r + 1}\left(\dfrac{1}{F_U}\right)} \tag{5}$$

where:

r is the number of failures observed
n is the sample size

F_U corresponds to the F distribution for the following degrees of freedom and associated required CL

$$\nu_1 = 2\,(r + 1)$$
$$\nu_2 = 2\,(n - r)$$

Tables of the values of the F distribution can be found in statistics textbooks. Using the example where 38 parts were tested and two failures occurred, the proportion defective of the population can be estimated with 90% confidence by using Equations 4 and 5. We find that $F_L = 3.79$ and $p_L = 0.014$, and that $F_U = 3.06$ and $p_U = 0.203$. We can, therefore, state with 90% confidence that the true p of the population lies between 1.4% and 20%. To narrow the range, we must test a larger sample or accept a lower Confidence Level. Of course, if we observed fewer failures, the range would also be smaller.

Tables Available on RAC Site

Given a desired confidence level, Equation 2 can be used to determine the sample size n given r defects via a trial and error approach. The value of n is varied until the desired Confidence Level is reached. Computer spreadsheets, e.g., Excel, can be used to make the calculations. Note that Excel has a limitation in that the largest factorial that it can use is 170!. If you need to calculate factorials of numbers larger than 170, RAC recommends you solve the binomial equation using logarithms.

RAC recognized that calculations using the Binomial distribution can be tedious, time consuming, and easily result in mistakes. Therefore, we developed a series of tables for different proportion defective LTPD: p = 0.01, 0.05, 0.10, 0.15, and 0.20 and a calculator. These are available on the RAC web site at <http://rac.iitri.org/Toolbox/>.

Tables **Calculator**

Table 2 shows the sample size required for a given number of failures to achieve a desired confidence level for p = 0.10. To use the table, assume the plan is to test 45 units and allow no failures as the criterion of success. Go to the row of the table with "0" under the column "Number of Failures" and read across to the right to the last column. The value is 45. One would be 99% confident that zero failures in a test of a sample of 45 items indicates that the population is no more than 10% defective. If two failures were allowed, you would go to the left column and find the value 2. A sample size of 45 lies between a 80% and 90% Confidence Level. By interpolation, the Confidence Level that the population is 10% defective or less is 83%.

Table 2. Sample Size Required for p = 0.1 To Achieve a Desired Confidence Level

No. of Failures	Confidence Levels				
	60%	80%	90%	95%	99%
	Sample Size				
0	9	16	22	29	45
1	20	29	38	47	65
2	31	42	52	63	83
3	41	55	65	77	98
4	52	67	78	92	113
5	63	78	91	104	128
6	73	90	104	116	142
7	84	101	116	129	158
8	95	112	128	143	170
9	105	124	140	156	184
10	115	135	152	168	197
11	125	146	164	179	210
12	135	157	176	191	223
13	146	169	187	203	236
14	156	178	198	217	250
15	167	189	210	228	264
16	177	200	223	239	278
17	188	211	234	252	289
18	198	223	245	264	301
19	208	233	256	276	315
20	218	244	267	288	327
22	241	266	290	313	342
24	262	286	312	340	378
26	282	308	330	364	395
28	303	331	354	385	430
30	319	354	377	408	448
35	374	403	430	462	505
40	414	432	490	512	565
45	478	510	550	580	620
50	513	534	595	628	675

The tables available from the RAC web site provides the user with a quick method of approximating:

- The sample size required to achieve a desired CL given an expected or allowable number of failures.
- The CL given the allowable number of failures and sample size.
- The allowable number of failures given the CL and the sample size.

For Further Study

a. O'Connor, P. D. T., "Practical Reliability Engineering," John Wiley & Sons, 1995.

b. John, P. W. M., "Statistical Methods in Engineering and Quality Assurance," John Wiley & Sons, 1990.

c. Juran, J. M. & F. M. Gryna, Jr., "Quality Planning & Analysis," McGraw-Hill, 1980.

d. Reliability Analysis Center, "Practical Statistical Tools for the Reliability Engineer," September 1999.

e. <http://vassun.vassar.edu/~lowry/binom_stats.html> (on-line Exact Binomial Probability Calculator)

f. <http://www.math.uah.edu/stat/bernoulli/bernoulli2.html> (The Binomial Distribution)

Other START Sheets Available

RAC's Selected Topics in Assurance Related Technologies (START) sheets are intended to get you started in knowledge of a particular subject of immediate interest in reliability, maintainability, supportability and quality.

94-1	ISO 9000
95-1	Plastic Encapsulated Microcircuits (PEMs)
95-2	Parts Management Plan
96-1	Creating Robust Designs
96-2	Impacts on Reliability of Recent Changes in DoD Acquisition Reform Policies
96-3	Reliability on the World Wide Web
97-1	Quality Function Deployment
97-2	Reliability Prediction
97-3	Reliability Design for Affordability
98-1	Information Analysis Centers
98-2	Cost as an Independent Variable (CAIV)
98-3	Applying Software Reliability Engineering (SRE) to Build Reliable Software
98-4	Commercial Off-the-Shelf Equipment and Non-Developmental Items
99-1	Single Process Initiative
99-2	Performance-Based Requirements (PBRs)
99-3	Reliability Growth
99-4	Accelerated Testing
99-5	Six-Sigma Programs
00-1	Sustained Maintenance Planning
00-2	Flexible Sustainment
00-3	Environmental Stress Screening

These START sheets are available on-line at <http://rac.iitri.org/DATA/START>.

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Future Issues

RAC's Selected Topics in Assurance Related Technologies (START) are intended to get you started in knowledge of a particular subject of immediate interest in reliability, maintainability, supportability and quality. Continuing with the subject of "One-Shot" devices, a future START sheet will cover Reliability Growth for "One-Shot" devices based on the Crow/AMSAA Discrete Model.

Please let us know if there are subjects you would like covered in future issues of START.

For further information on RAC START Sheets contact the:

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The Reliability Analysis Center is a Department of Defense Information Analysis Center (IAC). RAC serves as a government and industry focal point for efforts to improve the reliability, maintainability, supportability and quality of manufactured components and systems. To this end, RAC collects, analyzes, archives in computerized databases, and publishes data concerning the quality and reliability of equipments and systems, as well as the microcircuit, discrete semiconductor, and electromechanical and mechanical components that comprise them. RAC also evaluates and publishes information on engineering techniques and methods. Information is distributed through data compilations, application guides, data products and programs on computer media, public and private training courses, and consulting services. Located in Rome, NY, the Reliability Analysis Center is sponsored by the Defense Technical Information Center (DTIC). Since its inception in 1968, the RAC has been operated by IIT Research Institute (IITRI). Technical management of the RAC is provided by the U.S. Air Force's Research Laboratory Information Directorate (formerly Rome Laboratory).